Math Camp	**Summer 2018**

Lecture 2: August

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In this part of the course, we are going to cover basic topics of the set theory and formally and informally describe the functions.

2.1 Introduction to Set Theory

We are going to develop a working knowledge of the methods and ideas of set theory through an informal discussion. In mathematics, the meaning of the word "set" is used to represent a collection of objects viewed as a single entity. For example such nouns as "crowd ", "team", or "AGEC PH.D Students " are all examples of sets. The individual objects (player for the team) in the collection are called *elements* or *members* of the set, and they are said to *belong* in the set.
Usually sets are denoted by capital letter : $A, B, C, \ldots$ and elements are designated by lower-case letters: a,b,....

$$x \in S$$

means *x is an element of S*. For example, if $S = \{2, 4, 6\}$ then $2 \in S$

Example 1 *Designate the following sets*

1. $A = \{x|x^2 - 1 = 0\}$
2. $C = \{x|x + 8 = 9\}$

Definition 2.1 *A set A is is said to be a subset of a set B, whenever every element of A also belongs to B, or mathematically*

$$A \subseteq B \iff \forall a \in A \text{ then } a \in B$$

Note that the statement $A \subseteq B$ does not rule out possibility that $B \subseteq A$. In fact both happens only if A and B have the same elements. In other words,

$$A = B \iff A \subseteq B \text{ and } B \subseteq A$$

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Example 2 $A = \{3, 5, 6, 9\} = B = \{5, 3, 9, 6\}$

If $A \subseteq B$ but $A \neq B$,than we say A is a *proper subset* of B. i.e $A \subset B$

Example 3 *Let* $A = \{3, 2\}$ *,* $B = \{2, 3, 6\}$*,* $C = \{2, 3, 4, 9\}$*. Prove that*

1. $A \subset B$

[0]All errors are my own.

2. $B \not\subset C$

The notation

$$x|x \in SandxsatisfiesP$$

means we want the set of all elements x in S which satisfy property P.

Example 4 *Let $S = -2, -4, -9, 0, 1, 6, 8, 9$. Designate numbers in S that satisfy following property:*

1. *All positive*
2. *All even*

2.1.1 Unions, intersections, complements

From two sets A and B, we can form a new set called the *union* of A and B.

$$A \cup B$$

. That is, this is defined as the set of those elements which are in A, in B, or in both, In other word this is the set of all elements which belong to at least one of the sets A,B.

The same way the intersection

$$A \cap B$$

is the set of those elements common to both A and B. Two sets A and B are disjoint if $A \cup B = \emptyset$. The *difference* or complement of B relative to A of two sets is $A - B$ defined to be the set of all elements of A wjocj are not in B. In other words

$$A - B = \{x|x \in A \text{ and } x \notin B\}$$

Example 5 *Prove the following set relations*

1. *Commutative laws: $A \cap B = B \cap A, \;\; A \cup B = B \cup A$*
2. *If $A \subset C$ and $B \subset C$, then $A \cup B \subset C$*
3. *If $A \subset B$ and $B \subset C$, then $A \subset C$*

We also can have a sequence of sets $A_1, A_2, \ldots, A_n, \ldots$. For example

$$\bigcup_{i=1}^{n} A_i \equiv \{x|x \in A_i \text{ for some } i = 1, 2, \ldots, n\}$$

$$\bigcap_{i=1}^{n} A_i \equiv \{x|x \in A_i \text{ for each } i = 1, 2, \ldots, n\}$$

Definition 2.2 *Let A be a set. The power set of A, is the set of subsets of A, $\mathcal{P}(X) \equiv \{B|B \subseteq A\}$*

Example 6 *Find the power set for $C = \{2, 4, 6\}$*

Note that previously our sets were not *ordered*, that there were no difference were the specific element's place is. If we have pair of objects x and y and if we with to distinguish one of the objects, say x, as the *first* member and the other b, as the *second*, we enclode objects in paranthesis, (a, b).We refer to this as an *ordered pair*. The set of order pairs with the fist component in A and the second component in B, denoted $A \times B$, is the set of lists of two elements, the first in A and the second in B,taking into account the order of the list. For instance if $A = \{6, 4\}$, and $B = \{4, 5, 6\}$, then $(6, 4), (6, 5)$ are ordered pairs in $A \times B$.It is important to note that (4,6) is an ordered pair in $A \times B$ too, and it is different than $(6, 4)$. The set $A \times B$ is called the **Cartesian product** between A and B. Mathematically

$$A \times B \equiv \{y|y = (a, b), a \in A, b \in B\}$$

2.2 Functions

In everyday life we always deal with relationships that exist between one collection of objects and another. Graphs, charts and curves are devices for describing special relations in a quantitative fashion. Mathematicians refer to certain types of these relations as *fashions.*

Example 7 *The area of a square is a function of its edge-length. If the edge have length x, the area $A = x^2$.*

The word function was introduced into mathematics by Leibniz, however the meaning of the word has since undergone many stages of generalization. The **informal** definition of function is following

Definition 2.3 *(Informal) Given two sets A and B, a function is a correspondence which associates with each element of X one and only one element of Y.*

The set X is called the *domain* of the function. Those elements of Y associated with the elements in X form a set called the *range* of the function. (Note: This may be all of Y, but it not be.) If f is a given function and if x is an object of its domain, $f(x)$ designates the *value of f a x* or the *image of x under f*. The idea of function schematically can be illustrated as in Figure **??** In Figure **??** the collection X and Y are thought

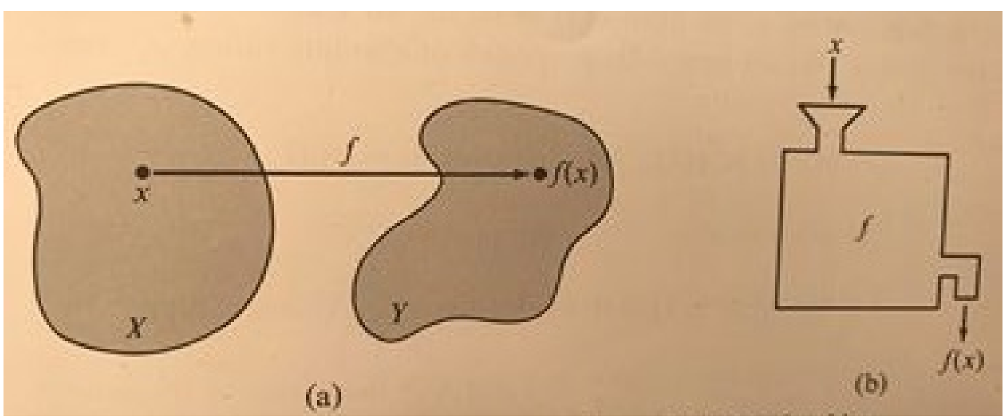

Figure 2.1: Schematic representation of the function idea. (Figure borrowed from [apostol1969].)

of as sets of points and an arrow is used to suggest a pairing of a typical point x in X with the image $f(x)$ in Y.

Example 8 1. *Identity function $f(x) = x$*

2. *The absolute value function* $f(x) = |x|$

Definition 2.4 *A function* f *is a set of ordered pairs* (x, y) *no two of which have the same first member. In other way, a function* f *from* X *to* Y *(*$f : X \to Y$*) is a subset of* $X \times Y$*, such that for each* $x \in X$*, there is a unique* $y \in Y$*.*

X called *domain* and the set$R \equiv \{y \in Y | \exists x \in X, y = f(x)\}$ is the *range* of f .

Definition 2.5 *Let* $f : X \to Y$

1. f *is said to be one-to-one or injective if and only if*

$$x_1, x_2 \in X \text{ and } f(x_1) = f(x_2) \text{ imply } x_1 = x_2$$

   or

$$x_1 \neq x_2 \text{ imply} f(x_1) \neq f(x_2)$$

2. f *is said to be onto or surjective if and only if*

$$\text{for each, } y \in Y \ \exists x \in X \text{ such that } y = f(x)$$

A $1 - 1$ onto functions called *bijactions.*

Example 9 *Consider* $f(x) = x^2$.

1. *Is it* $1 - 1$ *from* $[0, \infty) \to [0, infty)$
2. *what about from* $[0, \infty) \to$ *any open interval, which contains* 0

Note, if f is $1 - 1$ from a set X onto a set Y, we say that f *has an inverse function.* $f(f^{-1}(y)) = y$ We have not defined yet what we mean by $f(g(y))$.

Definition 2.6 *Given* $f : X \to Y$ *and* $g : Y \to Z$*,* $g \circ f : X \to Z$ *is defined by*

$$g \circ f(x) = g(f(x))$$

Example 10 *Prove that* $f(x) = e^x - e^{-x}$ *is* $1 - 1$ *on* $\mathcal{R}$*. Find the formula for* f^{-1}

2.3 References

References

[1] Apostol, T.M.. *Calculus.* Blaisdell Pub. Co., 1969.

[2] Wade, W.R. *An Introduction to Analysis* Pearson Education, 2004.